Volatility and the Cross-Section of Equity Returns: The Role of Short-Selling Constraints

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Abstract

A number of papers document a strong negative relation between idiosyncratic volatility and risk-adjusted stock returns. Using IHS Markit data on indicative borrowing fees, we show that stocks with high idiosyncratic volatility are far more likely to be hard-to-borrow than stocks with low idiosyncratic volatility. When hard-to-borrow stocks are excluded, the relation between idiosyncratic volatility and stock returns disappears. The relation between idiosyncratic volatility and stocks returns is more accurately described as a relation between being hard-to-borrow and stock returns.

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1. Introduction

Ang, Hodrick, Xing, and Zhang (2006) show that stocks with high idiosyncratic volatilities earn smaller risk-adjusted returns than stocks with low idiosyncratic volatilities. This finding has since been replicated by many other researchers. The inverse relation between idiosyncratic volatility (IVOL) and risk-adjusted returns is anomalous. Theory suggests that if investors hold diversified portfolios there should be no relation at all between idiosyncratic volatility and returns. If investors hold undiversified portfolios, the correlation between idiosyncratic volatility and returns should be positive, not negative.

In this paper, we examine the relation between short selling costs, idiosyncratic volatility and stock returns. We use indicative fees from HIS Markit to determine which stocks are hard-to-borrow. These fees are a direct measure of the difficulty of shorting a stock and are therefore preferable to other measures of short sale constraints like short interest or institutional holdings. We find that stocks with high IVOLs are far more likely to be hard-to-borrow than stocks with low IVOLs. Each month over 2003-2018, we sort stocks into quintile portfolios on the basis of idiosyncratic volatility. We confirm that there is a strong negative relation between idiosyncratic volatility and both returns and Fama-French-Carhart alphas when all stocks are included in the quintile portfolios. When we remove the hard-to-borrow stocks from the portfolios, returns of high idiosyncratic volatility portfolios exceed returns of low idiosyncratic volatility portfolios and there is no relation between idiosyncratic volatility and risk-adjusted returns.

In addition, we match individual stocks to form portfolios with nearly identical indicative short selling fees but large differences in idiosyncratic volatilities. Differences in four-factor alphas across these portfolios are small and insignificant. On the other hand, when we match individual stocks to form portfolios with nearly identical idiosyncratic volatilities but large differences in indicative fees, differences in alphas between low and high fee portfolios are large and highly significant.

Our evidence suggests that the relation between idiosyncratic volatility and four-factor alphas is really a relation between short-selling constraints and risk-adjusted returns. This is intuitively more plausible than a negative relation between volatility and returns. If stocks with high IVOLs are hard-to-borrow, they may adjust quickly to positive information but slowly to negative information. This could leave them overpriced on average. If high IVOL stocks are hard-to-borrow, it may not be possible to profit from the negative relation between idiosyncratic volatility and stock returns.

A related anomaly is the poor average returns earned by lottery stocks. Bali, Cakici, and Whitelaw (2011) show that stocks with large maximum daily returns over a month earn low average returns the following month. Stocks with large maximum returns are typically stocks with high idiosyncratic volatilities. When hard-to-borrow stocks are excluded, the poor performance of lottery stocks also disappears. The poor returns of lottery stocks are really the poor returns of hard-to-borrow stocks.

Our paper is related to Stambaugh, Yu, and Yuan (2015). They suggest that arbitrage is especially risky for high IVOL stocks, and that high IVOL stocks will tend to be both more overpriced and more underpriced than low IVOL stocks. They go on to say that the overall negative relation between IVOL and returns is due to short-sale constraints leaving more stocks overpriced than underpriced. This is different from of our paper in an important way. We find that it is not just that high IVOL stocks have arbitrage risk and that short selling constraints slow the arbitrage of overpriced stocks more than the arbitrage of underpriced stocks. There is a more direct relation. In general, high IVOL stocks are hard-to-borrow.

We replicate Stambaugh, Yu and Yuan's (2015) study for the sample period of 2003-2016, when both indicative fees and their underpricing variable are available. We show that when we omit hard-to-borrow stocks the relation between IVOL and returns disappears for Stambaugh, Yu, and Yuan's underpriced stocks. For high IVOL stocks, the difference in returns between stocks that Stambaugh, Yu and Yuan identify as under and overpriced disappears when hard-to-borrow stocks are excluded. For some of the other IVOL categories, however, underpriced stocks continue to outperform overpriced stocks.

The rest of the paper is organized as follows. Section 2 reviews the burgeoning literature on the relation between idiosyncratic volatility and stock returns. In section 3 we describe the data used here. In Section 4 we examine the relation between stock returns and idiosyncratic volatility after controlling for short selling restrictions. Section 5 examines the returns to lottery stocks in the context of short selling restrictions. Section 6 offers conclusions.

2. Literature review

2.1 Idiosyncratic volatility and stock returns

The inverse relation between idiosyncratic volatility and stock returns was brought to the attention of researchers by Ang, Hodrick, Xing, and Zhang (2006). They compute both idiosyncratic volatility and abnormal returns using the Fama-French three-factor model as a benchmark. They examine L/M/N strategies in which quintile portfolios are formed on the basis of idiosyncratic volatilities over an L month estimation period, M months are then skipped, and abnormal returns are calculated over the following N months. Most subsequent work examines 1/0/1 strategies, but Ang, Hodrick, Xin, and Zhang show that the low volatility quintiles earns three-factor abnormal returns that are significantly larger than the abnormal returns earned by high volatility quintile portfolio regardless of whether 1/0/1, 1/1/1, 1/1/12, 12/1/1, 12/1/12 strategies are used. The abnormal returns are centered in the high idiosyncratic volatility portfolios. That is, the quintile portfolio of high idiosyncratic volatility stocks earns large statistically significant negative abnormal returns while the quintile of low volatility stocks earns modest, statistically insignificant positive abnormal returns.

Ang, Hodrick, Xin, and Zhang (2006) find that their results on idiosyncratic volatility and returns are robust to controlling for a number of factors including leverage, liquidity, turnover, bid-ask spreads and dispersion of analyst forecasts. It is interesting that the relation between idiosyncratic volatility and abnormal returns is not statistically significant for the smallest quintile of firms, but is significant for portfolios formed from firms in larger size quintiles. The inverse relation between idiosyncratic volatility and returns persists in bull and bear markets, in NBER recessions and expansions and in stable and volatile markets.

Bali and Cakici (2008) examine the robustness of the inverse relation between idiosyncratic volatility and returns and document that the strength of the relation depends on the way in which portfolios are formed and idiosyncratic volatility is estimated. Using data from 1963-2004, they form quintile portfolios based in various ways on idiosyncratic volatility and compare returns across portfolios. In general, results are stronger when the breakpoints of portfolios are based on the distribution of volatilities across all stocks rather than the distribution across NYSE stocks. When all stocks are used to derive breakpoints, the stocks in the high idiosyncratic volatility portfolio typically have very small market capitalizations. Results are also stronger when idiosyncratic volatilities are estimated using daily returns over the previous month than monthly returns over the previous 24-60 months. Differences in returns between low and high idiosyncratic volatility portfolios are generally smaller and less statistically significant than differences in Fama-French three factor alphas. Differences are greater if the idiosyncratic volatility portfolios are value-weighted rather than equal-weighted. Bali and Cakici conclude that the inverse relation between idiosyncratic volatility and returns is sensitive to the way in which volatility is estimated and portfolio breakpoints are determined.

Ang, Hodrick, Xing, and Zhang (2009) study the relationship between idiosyncratic volatility (IVOL) and stock returns across developed country markets. They find that the low risk-adjusted returns are not just an American phenomenon but are, in fact a characteristic of stocks in each of the G7 countries and across the 23 countries that make up the MSCI Developed Country Index. There are large, significant comovements between the idiosyncratic volatility portfolio returns in other countries and in the U.S.

Ang, Hodrick, Xing, and Zhang (2009) also examine whether idiosyncratic volatility is a proxy for other stock characteristics. Using U.S. stocks, they run Fama-MacBeth regressions with excess returns as dependent variables and the previous month's Fama-French factor idiosyncratic volatility, the Fama-

French factors and stock characteristics as explanatory variables. The stock characteristics include the PIN measure of private information from Easley, Hvidkajer, and O'Hara (2002), the percentage of daily returns that are zero which proxies for trading costs (See Lesmond, Ogden, and Trzcinka (1999)), the number of analysts covering the stock, international ownership, the measure of delay in information incorporation from Hou and Moskowitz (2005), and skewness. A negative and significant relation between returns and idiosyncratic volatility remains when any or all of these variables are included in the Fama-MacBeth regressions.

Detzel, Duarte, Kamara, Siegel, and Sun (2019) show that the inverse relation between idiosyncratic volatility and abnormal returns that Ang et al. (2006) document over 1963-2000 also occurs before, in 1926-1963, and after, from 2001 – 2016. If idiosyncratic volatility portfolios are formed using the FF3 model and abnormal returns are calculated using more recent models from Hou, Xue, and Zhang (2015) or Stambaugh and Yuan (2017) or Barillas and Shanken (2018), differences in returns between high and low idiosyncratic volatility portfolios are no longer significant. The returns remain statistically significant, however, if these newer models are used to form idiosyncratic volatility portfolios and used to evaluate performance.

Bali, Cakici, and Whitelaw (2011) provide a behavioral explanation for the inverse relation between IVOL and stock returns. They cite evidence that investors prefer stocks with lottery-like payoffs. If investors prefer lottery stocks, the prices of stocks that offer the chance of a large payoff will be high and their average returns will be low. Bali, Cakici, and Whitelaw use the maximum daily return of a stock (MAX) over a month as a measure of its appeal to lottery stock investors. They sort stocks into decile portfolios on the basis of MAX and find that stocks with high maximum returns perform poorly on average over the next month. This result is robust to adjustment for other stock characteristics and with respect to variations in the way MAX is calculated.

Bali, Cakici, and Whitelaw (2011) observe that their measure MAX is correlated with idiosyncratic volatility. They find that after adjusting for MAX, the impact of idiosyncratic volatility on the returns of value-weighted portfolios is much reduced. For equal-weighted portfolios, controlling for MAX reverses the sign of the idiosyncratic volatility effect. High idiosyncratic volatility stocks earn greater returns than stocks with low idiosyncratic volatility.

Boyer, Mitton, and Vorkink (2010) explore a similar explanation for the relation between idiosyncratic volatility and returns. They observe that if investors prefer positive skewness, which is similar to preferring lottery stocks, idiosyncratic skewness should be associated with lower returns. As Boyer, Mitton, and Vorkink note, however, idiosyncratic skewness is hard to estimate because it is, by definition, a result of small probability events. They show that idiosyncratic volatility does a better job of predicting future idiosyncratic skewness than does current idiosyncratic skewness. Boyer, Mitton, and

Vorkink show that despite the close relation between the variables, there is still a negative relation between returns and idiosyncratic volatility after adjusting for idiosyncratic skewness.

Chen and Petkova (2012) note that idiosyncratic risk is defined relative to a model, generally the Fama-French three-factor model. If idiosyncratic risk is correlated with a missing risk factor, it may predict abnormal returns. Chen and Petkova break down changes in market volatility into changes in the average stock variance and changes in the average correlation of stock returns. Returns of portfolios with high idiosyncratic volatility have positive correlations with changes in average stock variance. In cross-sectional regressions that include loadings on the Fama-French factors and on changes in average stock variance and average stock correlation, idiosyncratic volatility is not significantly related to stock returns.

Fu (2009) observes that asset pricing models that assume investors under-diversify predict that stocks with high idiosyncratic volatility should have high expected returns. This makes the finding of an inverse relation between idiosyncratic volatility and stock returns particularly surprising. Fu notes that observed idiosyncratic volatility in one month may be a poor proxy for expected idiosyncratic volatility in the following month. He estimates expected idiosyncratic volatility with an EGARCH model and finds that it is associated with higher, not lower returns in the following month. Fu suggests that firms with high idiosyncratic volatility in one month are likely to have earned large positive returns that are reversed in the following month.

Stambaugh, Yu and Yuan (2015) suggest that arbitrage asymmetry is behind the negative relation between idiosyncratic volatility and stock returns. They note that idiosyncratic volatility represents risk to arbitrageurs who hold unbalanced portfolios while attempting to profit from mispricing. This limits the size of arbitrage positions and slows the adjustment of stock prices. Among overpriced stocks, those with the highest idiosyncratic volatility should be most overpriced. Among underpriced stocks, those with the highest idiosyncratic volatility should be most underpriced. Short sale restrictions, and the fact that a greater amount of arbitrage capital is devoted to long positions than short positions suggest though that mispricing is greater among overpriced stocks. As a result of this arbitrage asymmetry, there is a negative relation between idiosyncratic volatility and stock returns when all stocks are considered.

Stambaugh, Yu and Yuan (2015) construct a mispricing measure based on 11 return anomalies. When they double-sort stocks into portfolios based on this measure and idiosyncratic volatility, they find that the overpriced stocks with the most negative abnormal returns are those with high volatilities while the underpriced stocks with the largest positive abnormal returns also have the highest volatilities. Consistent with arbitrage asymmetry though, they find that the difference in returns between high and low idiosyncratic volatility overpriced stocks is much greater than the difference in returns between high and low volatility underpriced stocks.

Hou and Loh (2016) observe that there are certain types of stocks for which the relation between IVOL and returns is particularly strong. These include stocks with low institutional ownership and stocks with high short interest. Hou and Loh use a novel methodology to see how much of the relation between idiosyncratic volatility and returns can be explained by various factors. They divide factors said to explain the volatility-return relation into three groups: lottery preference variables, market friction variables, and others. Hou and Lou use five variables associated with lottery preferences: skewness, coskewness, expected idiosyncratic skewness, the maximum daily return over the month, and retail proportion of trades. For market friction, they use the lagged return, the Amihud illiquidity measure, the average bid-ask spread, and the number of zero returns over the month. Other variables include dispersion of analyst forecasts, standardized unexpected earnings, and the stock's exposure to the average variance component of the market variance. Hou and Loh use a four-stage methodology. First, they regress returns on IVOL. Second, they regress returns on both IVOL and an explanatory variable. In stage three they regress IVOL on the explanatory variable. They then decompose IVOL into a part directly related to the variable and an orthogonal part (intercept and error). In stage four, the coefficient from stage one is split into two coefficients: the one on the part of IVOL that is explained by the variable and on the part that is not explained by the variable. In general, the variables have explanatory power but only capture 10% to 30% of the relation between returns and idiosyncratic volatility. An exception is the maximum return variable, but the correlation between that and IVOL is about 0.89.

Han and Lesmond (2011) note that microstructure effects have a significant impact on estimates of idiosyncratic volatility. Bid-ask bounce from fluctuations between trades on bid prices and trades on ask prices inflates idiosyncratic volatility. Zero returns, which can occur if true returns are small relative to the bid-ask spread, may lead to an underestimate of idiosyncratic volatility. Using data from 1984 through June 2008, Han and Lesmond estimate idiosyncratic volatility from CRSP closing prices and from closing bid-ask midpoints. The difference in returns between low and high IVOL portfolios is much reduced in both magnitude and significance when the portfolios are formed on the basis of volatilities from bid-ask midpoint returns. Han and Lesmond also estimate residual idiosyncratic volatility by regressing volatility on the percentage of returns that are zero, the bid-ask spread, the bid-ask spread squared and the interaction between the spread and percentage of zero returns. When residual idiosyncratic volatility is used to form portfolios, the difference in returns is diminished and the difference in four-factor alphas is no longer significant.

2.2 Short sale constraints and stock returns

This paper shows that the relation between idiosyncratic volatility and stock returns is greatly diminished or disappears entirely after removing hard to borrow stocks. The market for stock borrowing is described and characterized by D'Avolio (2002). He studies the market for lending stock using daily data for April 2000 through September 2001 from one of the largest securities lenders in the world. He shows that about 16% of the stocks in CRSP were potentially impossible to borrow during this period. These stocks are typically tiny and illiquid stocks that in total accounted for less than 1% of total market value. Of stocks that were lent, about 91% were "general collateral" and could be borrowed easily for a small fee. The other 9% were hard-to-borrow, or on special, and had mean average lending fees of 4.3% per annum. D'Avolio hypothesizes that the likelihood that a stock is on special increases with differences of opinion between holders of the stock and short sellers. Consistent with this, he shows that the likelihood that a stock is on special increases with the dispersion of analyst forecasts and turnover.

Several papers present evidence that stocks with short sale constraints underperform. Desai, Ramesh, Thiagarajan, and Balachandran (2002) examine the returns to short-sale constrained Nasdaq stocks over 1988 – 1994. They use the short interest ratio, that is the ratio of short interest to shares outstanding, to define short sale constrained stocks. They find that heavily shorted stocks underperform the four factor model by 0.76% to 1.13% per month. Asquith, Pathak, and Ritter (2005) use the short interest ratio as a measure of demand to short and institutional ownership as a measure of the supply of shares. They define short-sale constrained stocks as those in the highest percentile of short-interest ratios and the bottom third of institutional holdings. Using the four factor model, they find that on an equal-weighted basis these firms underperform by 215 basis points in the next month. Boehme, Danielsen, and Sorescu (2006) note that in the Miller (1977) model it is the combination of short-sale constraints and differences of opinion that result in stock overvaluation. Their ideal measure of short-sale constraints is stock lending fees. They do not have that measure for their entire sample period so they use an estimate of fees from relative short interest and its interaction with exchange traded options for the stock. Similarly, their ideal measure of divergence of opinion is the dispersion of analyst earnings forecasts in IBES. This data is only available for a subset of firms though, so they regress IBES dispersion on share turnover and volatility and use the coefficients to estimate dispersion for other firms as well. They show that stocks that are in the highest quartile of dispersion of opinion and in the highest quartile of short-sale constraints underperform by about 1.7% per month using the four-factor model.

Most researchers treat shorting costs as a friction that slows the movement of prices to lower levels and prevents traders from profiting from predictable low returns. Drechsler and Drechsler (2014) use the spread in returns between stocks in the highest decile and lowest decile of shorting fees as a risk factor. When they include this with the Fama-French-Carhart four factors, they are able to reduce monthly abnormal returns to several anomalies, including IVOL, to statistical insignificance. It seems likely that individual stocks' sensitivity to this factor shifts significantly over time. They sort stocks into four fee buckets and find that after one year, only 45% of the high fees stocks are still in the high fee bucket. Over 20% have moved to the lowest fee bucket.

In D'Avolio (2002), differences in opinion about a stock's value increase the likelihood that the stocks will be on special. To the extent that differences in opinion are reflected in IVOL, this could be interpreted to mean that high IVOL is a cause of being hard-to-borrow. It is also possible though that being hard-to-borrow is a cause of stock return volatility. Diamond and Verrecchia (1987) model the impact of constraints on short selling on the incorporation of private information into stock prices. In their rational expectations model, investors and a market maker are aware that short-sales are constrained and stocks are not overpriced on average. They show, however, that it takes longer for stock prices to adjust to both positive and negative private information. On average, the absolute value of returns will be larger when public information is revealed if short sales are restricted. This suggests a larger volatility when a stock cannot be shorted.

3. Data

Data on daily stock returns, market capitalizations and stock turnover are obtained from CRSP. For each month over 2003-2018, we regress each stock's daily returns on the Fama-French factors. That is,

$$R_{it} = \alpha_{it} + \beta_i R_{mkt,t} + h_i HML_t + s_i SMB_t + \varepsilon_{it}$$

where R_{ir} is the returns of stock i on day t, $R_{Mkt,t}$ is the return on the market portfolio, HML_t is the return to the value factor for day t and SMB_t is return to the size factor on day t. The idiosyncratic variance, IVOL, of stock i for the month is $Var(\varepsilon_{it})$. For each month t, stocks are sorted into quintiles on the basis of their idiosyncratic volatility (IVOL) in month t-1. As was done in the previous literature, we eliminate stocks with the prices below $1 in month t-1.

Stocks are sorted into five IVOL portfolios each month. Panel A of Table 1 reports the time series means of each of these IVOLs. The low IVOL portfolio has a mean IVOL of 0.0086, indicating a standard deviation of return residuals of 0.86% per day. For the high IVOL portfolio, the mean daily standard deviation of return residuals is 5.40%.

Data on indicative stock borrowing fees for 2003-2018 are obtained from Markit. Indicative fees are the buyside rates, or what hedge funds and other short sellers would pay to borrow stocks. Markit obtains them daily from securities lending desks of over 100 major lenders and broker dealers. Daily data includes average fees, number of shares available for lending, and number of shares actually lent.

Indicative fees are missing for many small, illiquid stocks. According to Markit officials, these are stocks with no active borrowing market – typically because no investors are offering to lend the shares. This information led us to classify stocks with missing fees as hard-to-borrow. To the best of our knowledge, we are the first to do so. Within each IVOL quintile each month, we calculate the proportion of all stocks that have missing indicative fees and then average the proportions over months from 2003 through 2018. Panel A of Table 1 reports the average proportion of stocks with missing fees in each idiosyncratic volatility portfolio. The average proportion increases monotonically from 16.39% in the low idiosyncratic volatility portfolio to 33.60% of stocks in the high volatility portfolio. These proportions are higher than the 16% of stocks that D'Avolio (2002) says are impossible to borrow, but the proportions here are the average percentages with no lending market in a specific month rather than over a longer period.

For each stock, we use a simple average of daily indicative fees as the stock's monthly indicative fee. Each month, we classify stocks as high fee stocks if their mean indicative fee is among the highest 20% across all stocks with indicative fees. For each idiosyncratic volatility portfolio, we calculate the proportion of stocks that have high fees each month, and then calculate the average proportion over all months. The next column reports the time-series average proportion of stocks with high fees. This proportion increases monotonically from 6.31% of the low IVOL stocks to 41.38% of the high idiosyncratic volatility stocks. The next column of the table shows the proportion of stocks that have either high or missing indicative fees. These are the stocks that we define to be hard-to-borrow. This proportion is *not* the sum of the previous two columns because the first is the proportion of *all* stocks with missing fees while the second is the proportion of stocks *with fees* that are hard-to-borrow. The percentages in this column can be thought of as the proportion of stocks that are short-sale constrained. There is a clear and strong relation between idiosyncratic volatility and being hard-to-borrow. The proportion of hard-to-borrow stocks increases monotonically from 21.72% for the low IVOL quintile to 62.45% for the high IVOL quintile. Stambaugh, Yu and Yuan (2015) observe that high IVOL is associated with arbitrage risk and that short-sale constraints will slow price adjustment for overpriced stocks relative to that of underpriced stocks. Panel A of Table 1 shows that there is a direct relation between IVOL and being hard-to-borrow.

The last column in Panel A reports the mean size, or equity market capitalization for stocks in each quintile. There is an inverse relation between volatility and size. The mean firm size for stocks in the low idiosyncratic volatility portfolio is \$12.2 billion, while the mean firm size for high idiosyncratic volatility stocks is about 1/25th of that at \$513 million. There are a few reasons why idiosyncratic volatility may be lower for large firms. One is that large firms are more likely to have diversified businesses than small firms. In addition, firms with large equity capitalizations are less likely to be highly levered. Finally, microstructure noise is likely to contribute more to the idiosyncratic volatility of small, illiquid firms with low priced stocks than to the idiosyncratic volatility of large firms.

Panel B shows the percentage of stocks with missing indicative fees and the percentage of stocks that are hard to borrow for quintile portfolios formed on the basis of total volatility rather than idiosyncratic volatility. Results are similar to those in Panel A. The percentage of stocks with missing fee data and the percentage of stocks that are hard to borrow both increase with total volatility. Mean firm size decreases with total volatility.

Panel C of Table 1 describes the distribution of indicative fees for each year of the 2003-2018 sample period. Over this time, the total number of common stocks on the NYSE, Amex and Nasdaq decreased from 5,129 to 3,790. This decrease in the number of public companies has been documented and discussed elsewhere. Over the same period, the percentage of stocks with indicative fees increased from 62.7% to 97.3%. The indicative fees are for one-day loans, but are expressed as an annual interest rate. The 1st percentile of indicative fees declined slightly from 0.25% to 0.23% over 2003-2018, while the median indicative fee fell from 0.50% to 0.33% over the same period. In contrast, the mean indicative fee increases from 0.70% in 2003 to 4.07% in 2018. The change in mean fees is driven by increases in the fees of hard-to-borrow stocks. Fees for stocks at the 80th percentile increase from 0.63% in 2003 to 2.42% in 2018. Fees for stocks at the 99th percentile increase from 6.00% in 2003 to 90.30% in 2018. The increase in the indicative fees for the hard to borrow stocks occurs simultaneously with the increase in the proportion of stocks with indicative fees. Stocks that were not among those with indicative fees in one year turn out to have high fees when Markit includes them the following year. This is consistent with Markit's claim that stocks without indicative fees have little or no lending market and are hard-to-borrow.

To show this more explicitly, Panel D of Table 1 reports the distribution of indicative fees each year for the stocks that show up in the Markit data for the first time that year. Markit data starts in 2003, so all stocks with Markit data are new that year. For later years, the new stocks are either stocks that went public that year or stocks without an active lending market prior to that year. From 2005 on, median fees for new stocks are higher than the 80th percentile of fees for all stocks as shown in Panel B. In other words, most stocks with indicative fees for the first time are hard-to-borrow. If stocks with indicative fee data for the first time have such high fees, it seems highly likely that they were hard-to-borrow the previous year when fees are unavailable. We believe this justifies our decision to categorize stocks with missing fees as hard-to-borrow.

We use several other variables besides IVOL and stock borrowing fees to explain stock returns. To calculate market-to-book ratios (MB) we divide market capitalizations from CRSP by quarterly book values from Compustat. The book value of equity is defined as common equity plus balance sheet deferred taxes. As in Fama and French (1993) we exclude stocks with negative book values. Institutional holdings (INST) are obtained from Wharton Research Data Services' Thomson Reuters Institutional Stock Ownership Summary from 13F filings. Dispersion of analyst earnings forecasts is estimated, similar to Diether et al. (2002), as the standard deviation of analysts' current fiscal year earnings per share forecasts (ADisp) from IBES. As in Johnson (2004) and Nagel (2005), we scale the standard deviation by each firm's total assets to make the numbers comparable across firms. Turnover is the monthly share volume from CRSP divided by the number of shares outstanding.

Panel E of Table 1 provides means, standard deviations, and average number of observations per month for each of these variables. Panel F reports correlations of the variables. An unsurprising finding is that the correlation between idiosyncratic volatility (IVOL) and total volatility (TVOL) is very high at 0.971. IVOL is also negatively correlated with institutional holdings and firm size and positively related to indicative fees and turnover. Indicative stock borrowing fees are negatively correlated with institutional holdings and firm size.

4. Idiosyncratic volatility, stock borrowing fess and stock returns

4.1 The relation between idiosyncratic volatility and returns with and without hard-to-borrow stocks

Equal and value-weighted returns and Fama-French-Carhart four factor abnormal returns are calculated for each idiosyncratic volatility portfolio each month over 2003-2018. Time series averages of the returns and alphas are reported in Table 2, along with t-statistics that test whether the alphas are different from zero. All t-statistics are based on Newey West standard errors adjusted for three lags.

Results for equal-weighted portfolios are provided in Panel A. The first two columns report results when all stocks are used to form the idiosyncratic volatility portfolios. The average raw return of the low volatility portfolio is 94.2 basis points per month, while the average raw return of the high volatility portfolio is only 41.3 basis points. The difference is very large – over 50 basis points per month. Four factor alphas (Fama-French three factors plus momentum) are shown in the next column. The low IVOL portfolio has an average monthly alpha of 28.4 basis points. The t-statistic of 5.82 indicates that the alpha is significantly different from zero at any conventional significance level. The high IVOL portfolio has an alpha of -58.4 basis point per month. The average monthly difference in alphas is -86.8 basis points. The t-statistic for that difference is -3.84. So, in Table 2 we replicate the puzzling finding of Ang, Hodrick, Xing, and Zhang (2006) and others using our 2003-2018 sample period. High IVOL stocks underperform low IVOL stocks by an economically and statistically significant amount.

In the next two columns of Panel A, we show returns and alphas for the same idiosyncratic volatility portfolios after removing stocks with no indicative fee coverage in Markit. These stocks are disproportionately concentrated in the high IVOL portfolio. The mean raw return for the low IVOL portfolio changes slightly from 94.2 to 92.6 basis points per month, and the mean alpha goes from 28.4 to 26.5 basis points per month. In contrast, the mean raw return for the high IVOL portfolio jumps from 41.3 to 87.4 basis points per month and the alpha increases sharply from -58.4 to -14.1 basis points per month. The difference in returns between high and low IVOL portfolios almost disappears and is now only 5.2 basis points. The mean difference in alphas between the high and low IVOL portfolios is now -40.6 basis points with a t-statistic of -1.98.

Omitting stocks with no indicative fee coverage still leaves stocks that can be borrowed but are expensive to borrow. In the next two columns of Panel A, we show returns after omitting both stocks with missing indicative fees and stocks with indicative fees that are among the highest 20%. When we remove the complete set of stocks that are difficult to short, the inverse relation between IVOL and returns is completely eliminated. Now the quintile portfolio of stocks with the highest IVOL earns average returns of 1.235% per month, *more* than the 94.2 basis point per month average return of the quintile of stocks with the lowest IVOLs. The four-factor alpha for the low IVOL portfolio is now 27.7 basis points per month while the alpha for the high IVOL portfolio is 22.2 basis points per month. The difference, -5.5 basis points, is insignificant with a t-statistic of just -0.31. When hard-to-borrow stocks are removed, the relation between return and idiosyncratic volatility disappears.

The last two columns of Panel A show returns and four-factor alphas for the idiosyncratic volatility portfolios when only hard-to-borrow stocks from the original portfolios are included. Portfolios have unequal numbers of stocks, with high idiosyncratic volatility portfolios having more that are hard-to-borrow. For each idiosyncratic volatility portfolio, returns of hard-to-borrow stocks are lower than when all stocks are included. For all but the low volatility portfolio, four factor alphas are lower for the hard-to-borrow stocks than for all stocks. Using just the hard-to-borrow stocks, there appears to be a strong relation between idiosyncratic volatility and alphas. The hard-to-borrow stocks in the high volatility portfolio perform especially poorly. The high idiosyncratic volatility hard-to-borrow stocks have a monthly return of -19.1 basis points and a monthly alpha of -1.193%. When all stocks are included, the high volatility portfolio has a mean monthly return of 41.3 basis points and a mean alpha of -58.4 basis points. The difference in alphas goes from -0.868% for all stocks to -1.563% when only the hard to borrow stocks are included.

Panel A of Table 2 seems to indicate that there is a relation between returns and idiosyncratic volatility, but only for hard-to-borrow stocks. We will show later that some of the hard-to-borrow stocks are harder-to-borrow than others. Among the hard-to-borrow stocks, indicative fees are much higher for high IVOL stocks than for others. This suggests that the relation between IVOL and returns among hard-to-borrow stocks is really a relation between the size of borrowing fees and returns.

Panel B repeats the analysis of Panel A with value-weighted portfolios. With these portfolios, differences in returns and alphas of high and low IVOL portfolios are smaller. When all stocks are included in the portfolios, the average raw return for the high IVOL portfolio is just 11.6 basis points per month less than the average raw return of the low IVOL portfolio. The difference in alphas is larger with high IVOL portfolio having monthly alphas that average 60.7 basis points less than the alphas of the low IVOL portfolio. The t-statistic for this difference is -2.28.

As with equal-weighted portfolios, omitting stocks with missing indicative fees and with indicative fees that are among the highest 20% eliminates the inverse relation between IVOL and portfolio performance. With these stocks omitted, the average raw return of the high IVOL portfolio is actually 27.5 basis points greater than the average raw return of the low IVOL portfolio. The four factor alpha for the high IVOL portfolio is 18.4 basis points less than the alpha for the low IVOL portfolio, but this difference is insignificant, with a t-statistic of just -0.69. With value-weighted portfolios, as with equal-weighted portfolios, the relation between idiosyncratic volatility and returns disappears when hard-to-borrow stocks are removed.

The last two columns of Panel B provide returns and four-factor alphas for idiosyncratic volatility portfolios composed only of hard-to-borrow stocks. Alphas for all portfolios are now negative, but they are especially low for the high volatility portfolio. The mean return is -22.2 basis points per month and the mean alpha is -1.449% for the high idiosyncratic volatility portfolio. The difference in alphas between high and low IVOL portfolios is -1.335% with a t-statistic of -3.93.

Ang, Hodrick, Xing, and Zhang (2006) also form portfolios based on total volatility and find that stocks with high total volatility, like stocks with high idiosyncratic volatility, earn negative abnormal returns. Table 3 is similar to Table 2, but provides returns and four-factor alphas for quintile portfolios formed on the basis of total volatility rather than idiosyncratic volatility. Panel A reports results for equal-weighted portfolios. When all stocks are included, the low total volatility portfolio has a mean return of 94.9 basis points per month while the high volatility portfolio has a mean return of 45.7 basis points per month. The difference in four-factor alphas is larger. The mean monthly alpha for the low volatility portfolio is 36.7 basis points while the mean alpha of the high volatility portfolio is -63 basis points per month. The difference of -99.7 basis points per month is highly significant with a t-statistic of -4.80. When stocks with missing indicative fees and stocks with high borrowing fees are omitted, the four-factor alpha for the high volatility quintile goes from -63 basis points per month to 12.1 basis points per month. The difference between the average alpha of the high volatility portfolio and the low volatility portfolio goes from -99.7 basis points per month to -23.8 basis points per month. After taking out the hard-to-borrow stocks, the relation between total volatility and abnormal returns is insignificant.

It is interesting that when all stocks are included the mean monthly raw return of the high total volatility portfolio is 45.7 basis points while the mean raw return of the low total volatility portfolio is 94.9 basis points. When the stocks with missing fees and the stocks that are hard-to-borrow are removed, the difference in raw returns changes sign. The mean monthly raw return of the low total volatility portfolio is almost unchanged at 94.6 basis points while the mean raw return of the high total volatility portfolio is now higher at 1.214%.

The last two columns of Panel A report returns and alphas for hard-to-borrow stocks in each of the total volatility portfolios. The greater the total volatility, the lower the return and alpha of hard-to-borrow stocks. For the lowest total volatility portfolio, the mean monthly return is 87.8 basis points and the mean monthly alpha is 38.7 basis points. For the high total volatility portfolio, the mean monthly return is -22.1 basis points and the mean alpha is -1.292%. The difference in alphas is a highly significant 1.679% per month. Among stocks that are not hard-to-borrow, there is no relation between total volatility and returns. Among stocks that are hard-to-borrow, there is a strong relation between total volatility and returns. But, among the hard-to-borrow stocks, the high total volatility stocks are much more expensive to borrow than the low total volatility stocks.

Panel B of Table 3 shows results for value-weighted portfolios. When all stocks are sorted into value-weighted quintile portfolios on the basis of total volatility, there is little difference in the raw returns of high and low volatility stocks. Four-actor alphas do, however, differ significantly across volatility quintiles. Low volatility stocks have an average monthly alpha of 23.6 basis points while high volatility stocks have an average monthly alpha of -46.9 basis points. The difference, -70.5 basis points, is statistically significant with a t-statistic of -2.54. When all hard-to-borrow stocks are eliminated, the difference in alphas falls to -46.7 basis points with a t-statistic of -1.68. When only hard-to-borrow stocks are considered, the difference in returns between low and high total volatility stocks is especially large, over 1% per month. The difference in four factor alphas is -1.706% per month.

The results in Tables 2 and 3 suggest that the relation between volatility and returns is greatly diminished or disappears entirely when hard-to-borrow stocks are removed. By themselves, these results do not explain why there is a relation between idiosyncratic volatility and returns, but they do indicate that short-sale constraints make it unlikely that investors can earn these abnormal returns in practice.

4.2 Robustness tests

Table 4 presents a series of robustness tests that demonstrate that the results presented in Table 2 are not sensitive to the way we define hard-to-borrow and occur throughout our sample period. Panel A reports returns and alphas of IVOL quintile portfolios for the subperiods of 2003-2009 and 2010-2018. The first four columns contain results when all stocks are included in the portfolios. In both subperiods, alphas of low IVOL portfolios are positive and alphas of high IVOL portfolios are negative. The difference between alphas of the high and low IVOL portfolios is statistically significant in both subperiods. The last four columns provide returns and alphas when hard to borrow stocks are omitted from the portfolios. Without the hard-to-borrow stocks, the differences between high and low IVOL alphas are small and statistically insignificant in both subperiods.

Panel B contains subperiod results for value-weighted portfolios. When all stocks are included in the portfolios, the difference in alphas between high and low IVOL portfolios is -0.501 with a t-statistic of -1.48 over 2003-2009, and -0.562 with a t-statistic of -1.81 over 2010-2018. When the hard-to-borrow stocks are omitted, the differences in alphas shrink to -0.003 with a t-statistic of -0.01 for 2003-2009, and -0.173 with a t-statistic of -0.57 for 2010-2018. The results that we obtain for the entire period in Table 2 also hold for subperiods.

In Tables 2 and 3 we defined hard-to-borrow stocks are those with missing fees or fees that are among the highest 20% during that month. In Panel C of Table 4, we compare results using that definition of hard-to-borrow with a second definition in which stocks are hard-to-borrow if the indicative fee is missing or if it is at least three times as large as the median fee. When we define stocks as hard-to-borrow using the alternative measure and remove them from the IVOL portfolios, the differences in alphas between the high and low IVOL portfolios are also small and statistically insignificant. Our finding that the relation between IVOL and returns disappears when hard-to-borrow stocks are removed is not sensitive to our definition of hard-to-borrow.

The last two columns of Table 2 reported returns and alphas for the IVOL portfolios when only hard-to-borrow stocks were included. Using only hard-to-borrow stocks, we found a strong relation between IVOL and returns. In Panels D and E of Table 4, we examine how sensitive that finding is to our definition of hard-to-borrow. Panel D provides results for equal-weighted portfolios. The first two columns contain returns and alphas for the portfolios of hard-to-borrow stocks when hard-to-borrow is defined as in Table 2: missing indicative fees or having fees that are among the highest 20%. The difference in returns between high and low IVOL portfolios is -1.105% per month and the difference in Fama-French-Carhart alphas is -1.563% with a t-statistic of -6.12. The next two columns provide results when stocks are only counted as hard-to-borrow if indicative fees are missing for them. In this case, results are stronger. Now the high IVOL portfolio has a return of -1.151% for the month and the difference in returns between high and low IVOL portfolios is -2.046%. The difference in alphas is -2.448% with a t-statistic of -6.61. This could be interpreted to mean that stocks with missing fees are especially hard-to-borrow. The middle two columns report results when hard-to-borrow stocks are defined as just those with fees that are among the highest 20%. In this case the difference in returns of high and low IVOL portfolios is -0.684%, and the difference in alphas is -1.278% with a t-statistic of -

5.05. Results are not quite as strong as when both missing and high fee stocks are included in the hard-to-borrow portfolios, but using just high fee stocks generates a significant difference in returns between high and low IVOL portfolios. The last four columns of Panel D report results when high fee stocks are defined as those with indicative fees at least three times as large as the median. Results with this alternative definition of high fees are similar to but somewhat stronger than the original results, suggesting that this might be a better measure of high stock borrowing costs.

Panel E of Table 4 provides returns and alphas of value-weighted IVOL portfolios created using different definitions of hard-to-borrow. For value-weighted portfolios, like equal-weighted portfolios, the relation between IVOL and returns holds regardless of the exact definition of hard-to-borrow.

4.3 Separating the effects of IVOL and short sale constraints on returns

Within the set of stocks that are hard-to-borrow, there is an inverse relation between IVOL and four-factor alphas. But, there is also a relation between IVOL and the degree to which stocks are hard-to-borrow. Among the hard-to-borrow stocks, high IVOL stocks are harder to borrow than low IVOL stocks. To further explore how IVOL and borrowing costs interact to affect stock returns, we perform double sorts of stocks first into quintiles of idiosyncratic volatility and then into terciles of indicative fees. Note that we lose the stocks that are defined as hard-to-borrow because they are missing indicative fees, but retain hard to borrow stocks with high borrowing fees. We then calculate four factor alphas for each portfolio each month, and calculate averages across the months of 2003-2018.

Panel A of Table 5 reports results when stocks are equal-weighted in each portfolio. Within the low and middle indicative fee terciles, there is no relation between alphas and idiosyncratic volatility. In the highest indicative fee tercile, there is a strong and statistically significant inverse relation between idiosyncratic volatility and abnormal returns. The average monthly difference in alphas is -1.472%. Holding IVOL quintiles the same, high indicative fee stocks underperform low indicative fee only for the high IVOL quintile. The difference is -1.50% with a t-statistic of -5.97. Panel B shows results when returns in the portfolios of stocks sorted on both IVOL and indicative fees are value-weighted. Results are very similar to those of Panel A. High IVOL stocks underperform low IVOL stocks when and only when indicative fees are high. High indicative fee stocks underperform low indicative fees stocks, but the difference is only significant for the high IVOL quintile.

Table 6 helps to explain these findings. Panel A shows the mean indicative fee for each of these portfolios. There is no variation in indicative fees between low and high IVOL portfolios when indicative fees are low and only moderate variation across IVOL portfolios in the second indicative fee tercile. In the high indicative fee tercile, there is a big difference in fees between low and high IVOL portfolios. For the low IVOL portfolio, the mean indicative fee is 1.2% per year. In the high IVOL portfolios it is ten times as high at 12.5% per year. It is within the high indicative fee tercile, where there is a large spread in indicative fees across IVOL portfolios, that there is a significant difference in alphas across IVOL portfolios. Similarly, within the three lowest IVOL quintiles the differences in indicative fees between the high and low fee terciles are small, as are the differences in alphas. On the other hand, in the high IVOL quintile, the low indicative fee tercile has a mean indicative fee of just 40 basis points while mean indicative fee for the high fee tercile is 12.5%. It is within high IVOL quintile that the difference in alphas across indicative fee terciles is large and significant.

Panel B provides mean IVOLs for each of these portfolios. For each of the three idiosyncratic volatility terciles, there is a big spread in the IVOLs between the high and low IVOL quintiles as we would expect. In the high indicative fee tercile, the range of idiosyncratic volatilities is from 0.0084 to 0.0589. Even in the low indicative fee tercile the range is from 0.0086 to 0.0480. If differences in IVOL are behind the differences in abnormal returns in Table 5, we would expect significant differences in abnormal returns for each of the indicative fee terciles. But, that's not what we find. We only see significant differences in abnormal returns in the high fee tercile where there is a large difference in fees across IVOL portfolios. Likewise, if differences in IVOL were responsible for the differences in returns we would not expect to see large differences in returns across indicative fee terciles within an IVOL quintile. Again, that is not what we find. There are large and significant differences in alphas across indicative fee terciles for the high volatility quintile where there is also a large difference in indicative fees.[1]

There are two lessons to be drawn from Table 6. First, costs of shorting make it very hard to profit from the negative abnormal returns earned by high indicative fee, high IVOL stocks. Table 5 shows that the mean alpha for the high indicative fee-high IVOL portfolio is -1.1% per month for equal-weighted portfolio. Table 6 shows the mean indicative fee is 12.5% per year or a little over 1% per month for that portfolio. All of the alpha earned from shorting is eaten up by stock borrowing fees. Second, the relation between IVOL and returns appears to actually be a relation between shorting costs and returns. There is no relation between abnormal returns and IVOL for low and medium indicative fee stocks even though there is a lot of variation in IVOL within these fee categories. Table 5 shows that there is little variation in indicative fees across IVOL portfolios within the low and medium fee terciles, but a large spread in fees across IVOL quintiles for the high fee tercile. This is where high IVOL is associated with low returns.

[1] We also replicated our results using independent portfolio sorts on IVOL and indicative fee, and they are similar to those reported in the tables.

The results in Table 5, along with the characteristics of the portfolios in Table 6 suggest that it is difficulty in borrowing the stock, not a high IVOL that is associated with low alphas. Nevertheless, high IVOL stocks are often hard-to-borrow, so it is a challenge to separate the impact of IVOL and short selling constraints on stock returns. To get a cleaner separation of the two effects, we match firms on one characteristic while letting the other vary. First, for each month, we pick pairs of stocks with idiosyncratic volatilities that are within 1% of each other, and in which the indicative fee of one is at least twice as large as the other. So, for example, if the first stock in a pair has an idiosyncratic volatility of 2% and an indicative fee of 2%, a matching stock must have an idiosyncratic volatility between 1.98% and 2.02% and an indicative fee greater than 4% or less than 1%. The stock with the lower indicative fee is placed in the low fee portfolio and the stock with the higher indicative fee is placed in the high fee portfolio. Each stock is used as part of only one pair during a month. Portfolios are reformed each month, and equal-weighted returns and four-factor alphas are calculated for each succeeding month.

We follow the same procedure to assemble portfolios that match on indicative fees but differ on IVOLs. We find pairs of stock with indicative fees that are within 1% of each other but with one stock having an IVOL at least twice as large as the other. So, for example, if one stock with an idiosyncratic volatility of 1% and an indicative fees of 1%, the second must have an indicative fee between 0.99% and 1.01% and an IVOL that is less than 0.5% or more than 2%. The stock with the lower IVOL is placed in the low IVOL portfolio for that month while the stock with the higher IVOL is placed in the high IVOL portfolio that month.

Mean returns and alphas for these portfolios are reported in Table 7. Panel A provides results when one of the variables is within 1% across stocks and the other is at least twice as large for one of the stocks. The first three rows report means when idiosyncratic volatilities are within 1% of each other but indicative fees are very different. On average, over months from 2003 – 2018, the mean indicative fee is 6.535% for the high fee portfolio and 0.535% for the low fee portfolio. Mean IVOLs are very close; 3.030% for the low fee portfolio and 3.046% for the high fee portfolio. Mean returns are 1.076% for the low fee portfolio and 0.684% per month for the high fee portfolio. The difference of 39.2 basis points per month is statistically significant with a t-statistic of -3.44. The last column reports the four factor alphas for these portfolios. The mean alpha is 0.091% for the low fee portfolio and -0.208% for the high fee portfolio. The t-statistic for the difference is -2.72.

When we hold IVOL nearly constant and vary indicative fees, we find that the hard-to-borrow stocks underperform other stocks by a statistically and economically significant amount.

The next three rows of Panel A report mean returns and alphas for two portfolios with nearly identical indicative fees and very different IVOLs. The average indicative fee over all months from 2003 through 2018 is 1.079% for both the high and low IVOL portfolio. The mean IVOL in the low IVOL portfolio is 1.246% while the mean IVOL in the high IVOL portfolio is 3.293% per month. In this case, when indicative fees are held constant, we do not find that high IVOL stocks underperform low IVOL stocks. The mean return for high IVOL stocks is 1.113% per month and is actually higher than the mean return for low IVOL stocks of 0.958% per month. The t-statistic for this difference is just 0.88. The mean alpha for high IVOL stocks is 0.091% per month while the mean alpha for low IVOL stocks is 0.189%. This difference of -0.098% is not significant. The t-statistic is only -0.93.

When we hold indicative fee constant and vary IVOL, we find almost no difference in alphas and returns between high and low IVOL stocks. These results in Panel A suggest that it is indicative fees and not IVOL that is driving returns.

Panel B of Table 7 shows returns and alphas for matched firm portfolios when stricter matching criteria is used. Now the two stocks must differ by no more than 0.75% in indicative fees (IVOLs) and the IVOL (idiosyncratic fee) of one of the pairs must be at least three times as large as the other. Now, when the stocks are matched on idiosyncratic volatility, the mean difference in IVOLs is only 0.013% while the difference in indicative fees increase from 5.992% to 7.169%. With the bigger difference in indicative fees comes larger differences in returns and alphas. The return on the low fee portfolio now exceeds the return on the high fee portfolio by 0.444% per month. With the larger difference in fees, the mean four factor alpha of the low fee portfolio exceeds the mean four factor alpha of the high fee portfolio by 0.374% per month. Both differences are statistically significant at the 1% level.

The last three rows of Panel B compare returns and alphas for portfolios with near identical fees but different IVOLs. The mean IVOL for the low IVOL portfolio is now 1.049% while the mean IVOL for the high IVOL portfolio is 3.887%. Despite the large differences in IVOL, the differences in mean returns and mean alphas remain small and statistically insignificant.

4.4 The relation between IVOL and returns when multiple explanatory factors are included

We saw in Table 7 that when we formed portfolios with near equal IVOLs but different indicative fees, the high fee portfolio earned lower four-factor alphas the next month. In contrast, when we formed portfolios with near identical indicative fees but different IVOLs, the difference in alphas was insignificant. This suggests that the relation between IVOL and abnormal returns is really a relation between short-sale constraints and abnormal returns. We are, however, reluctant to draw such a strong conclusion. If we construct portfolios with the same IVOL but different indicative fees, we are not holding other things constant. The high indicative fee portfolio may differ from the low fee portfolio in firm size, market-to-book ratios, institutional holdings or other factors.

We examine the impact of IVOL and short sale constraints on abnormal returns after adjusting for other factors by running Fama-MacBeth regressions of stock excess returns on the previous month's idiosyncratic volatility and other variables. Cross-sectional regressions are run each month over 2003-2018, and time-series averages of the coefficients are calculated. We calculate Newey West t-statistics with adjustment for three lags using the time-series standard errors of the coefficients. An advantage of using regressions to assess the impact of idiosyncratic volatility on returns is that we can control for a number of other variables. A disadvantage of the regressions relative to the earlier comparison of returns across idiosyncratic volatility portfolios is that we assume a specific relation between volatility and returns. In these regressions we assume that the relation between idiosyncratic volatility and returns is linear.

Regression results are reported in Table 8. The first regression contains only idiosyncratic volatility as an explanatory variable. Its mean coefficient is -0.143 with a t-statistic of -3.06. Even though our paper covers a shorter and more recent period than most studies of idiosyncratic volatility, we find a negative and significant relation between volatility and stock returns. In regression (2) we include turnover, log of firm size, log of the market-to-book ratio, return the previous month and return the previous 12 months. As shown in Table 1, some of these variables, like turnover and log of firm size, are moderately correlated with idiosyncratic volatility. When these variables are included, the coefficient on idiosyncratic volatility shrinks to -0.085, but remains statistically significant.

Regression (3) includes the proportion of shares held by institutions and a dummy variable for stocks with missing institutional holdings. Institutional holdings is proxy for ease of short selling. Institutions are a source of shares for short sellers and stocks that are owned mainly by institutions are easier to borrow than stocks that are owned mainly by retail investors. When these variables are included in the regression, the coefficient on the dummy variable for missing institutional holding is negative and significant with a t-statistic of -5.91. The coefficient on idiosyncratic volatility goes from -0.085 to -0.066. The t-statistic is -1.65, indicating that idiosyncratic volatility is now of marginal statistical significance.

The fourth regression includes variables associated with differences in opinion across investors. We include ADisp, the dispersion of analyst earnings forecasts and a dummy variable that is one if analyst dispersion is missing. Both of these variables are significant, but the coefficient on idiosyncratic volatility also remains significant.

Regression (5) includes the indicative fee for stock borrowing and a dummy variable that equals one when indicative fees values are missing. Both of these variables are highly significant determinants of excess returns. The coefficient on the dummy for missing indicative fees is -0.013 with a t-statistic of -8.94, while the coefficient on indicative fees is -0.079 with t-statistic of -5.60. The coefficient on idiosyncratic volatility moves closer to zero at -0.056 with a t-statistic of just -1.41. When both factors are included in the regression, there is a strong relation between being hard-to-borrow and returns while the relation between idiosyncratic volatility and returns is insignificant.

Regression (6) includes the institutional investor and analyst dispersion variables, but does not include the indicative fee variables. The coefficient on idiosyncratic volatility is now -0.066 with a t-statistic of -1.66. When all variables are included in regressions (7), the coefficients on the dummy variable for missing fees, the dummy variable for missing institutional holdings, and indicative fees are all highly significant. The coefficient on idiosyncratic volatility is now just -0.053 with a t-statistic of -1.36. Table 8 reveals that the significant, negative relation between idiosyncratic volatility and returns is diminished and is no longer significant after including institutional holdings and especially after including indicative fees.

Our sample period, which is dictated by the availability of indicative fee data, is short. It is possible that IVOL would be statistically significant in the longer periods examined by others. Nevertheless, our findings indicate that short-sale restrictions are a much more powerful predictor of stock returns than IVOL and that IVOL's ability to predict stock returns is much reduced by also including short-sale restrictions in the regressions.

4.5 Options Order Imbalance and Idiosyncratic Volatility

If short sale constraints prevent investors from shorting stocks, they may instead use options to take bearish positions. We follow Hu (2014) and calculate the options order imbalance (OOI) for stocks each day by summing the products of the signed volume and option delta for all CBOE/ISE option trades on an underlying stock. We standardize each OOI by dividing by the shares outstanding. Unlike Hu (2014) we do not need to sign options volumes, as the exchanges provide end of day cumulative open buy, open sell, close buy, close sell non-market makers volumes for all call and put contracts. Open buy volume denotes opening new long option position, and open sell volume denotes opening a new short/written option position. Similarly close buy/sell denote closing previously open short/long positions. Combined, the aggregated options volumes across the two exchanges comprise about 60% of all option trading volume in the US. The advantage of using these data is that we can directly observe net imbalances of end-users. We then average daily OOI per month to estimate the monthly values. If most volume occurs from put purchases or call sales, OOI will be negative. If most volume comes from call purchases or put sales, OOI will be positive. Finally, we calculate the mean OOI for IVOL quintile portfolios of hard-to-borrow stocks and IVOL quintiles of stocks that are not hard-to-borrow. Results are shown in Table 9.

The first two columns of the table provide mean indicative fees and Mean OOIs for IVOL quintile portfolios composed of hard-to-borrow stocks. We include both stocks with missing fees and stocks with fees that are among the highest 20% in the hard-to-borrow stocks. As we go from the low IVOL portfolio to the high IVOL portfolio, mean indicative fees (for the stocks with fees) increase monotonically from 3.7% to 10.8%. The mean OOI falls from -0.0043% of shares outstanding for the low IVOL portfolio to -0.0143% of shares outstanding for the high IVOL portfolio. Recall that a negative OOI means that option traders are, on net, taking bearish negative delta option positions. The difference between high and low IVOL portfolio OOI's is highly statistically significant with a t-statistic of -6.21. In the next four columns, we look at portfolios of stocks with missing fees and portfolios of stocks with fees among the highest 20% separately. In both cases, the mean OOIs of the high IVOL portfolios are less than the mean OOIs of the low IVOL portfolios and the differences are statistically significant at the 5% level. Among hard-to-borrow stocks, high IVOL stocks appear to be especially hard-to-borrow. The large negative values of OOI for high IVOL stocks suggest that some investors attempt to circumvent the constraints on short-selling by instead taking negative delta positions in options.

Options market makers have to absorb most of these imbalances, and hence have higher inventory holdings of their options on stocks in the high IVOL portfolios. Ni, Pearson, Poteshman and White (2018) argue that option market makers' hedging of their inventories increases both stock volatility and the probability of large stock price moves. Market frictions like being hard-to-borrow can increase stock volatility through the channel of options market makers hedging activity, and investors trying to overcome high short-selling costs in the stock market by shorting in options. We know that there is a relation between being hard-to-borrow and having high idiosyncratic volatility. This suggests that it is possible that the causation runs from being hard-to-borrow to being volatile.

Results for stocks that are not hard-to-borrow are reported in the last two columns of Table 9. They are strikingly different. Now, as we go from low IVOL to high IVOL portfolios the mean OOI goes from negative to positive. When high IVOL stocks are not hard-to-borrow, investors, on average, take bullish positive delta options positions. The difference in mean OOIs between high and low IVOL portfolios is highly significant with a t-statistic of 14.40.

The results in Table 9 mirror those in Table 2 and provide additional indirect support for the hypothesis that the relation between IVOL and abnormal returns is really a relation between being-hard-to-borrow and abnormal returns. For hard-to-borrow stocks, and only for hard-to-borrow stocks, options investors make increasingly bearish negative-delta trades as we go from low to high IVOL stocks. For stocks that are not hard-to-borrow, investors make increasingly bullish positive-delta trades as we go from low to high IVOL stocks.

4.6 Idiosyncratic volatility, mispricing, and arbitrage risk

Stambaugh, Yu, and Yuan (2015) suggest that IVOL represents arbitrage risk that prevents investors from correcting mispricings. They provide evidence that high IVOL stocks can be either underpriced or overpriced more than low IVOL stocks. But, Stambaugh, Yu and Yuan point out that short selling costs and the restriction of many institutional investors to long positions means that overpricing will be corrected more slowly than underpricing leaving more high IVOL stocks overpriced than underpriced. Hence, as a whole there will be a negative relation between IVOL and returns. While Stambaugh, Yu, and Yuan recognize that there is a relation between mispricing and IVOL, and that short sale constraints may make overpriced stocks adjust to information more slowly than underpriced stocks, they do not appear to recognize that there is a direct relation between being hard-to-borrow and IVOL.

Stambaugh, Yu and Yuan (2015) construct a mispricing measure based on 11 return anomalies. Each month, they rank stocks by exposure to each of the anomalies, assigning higher ranks to the stocks that the anomaly indicates are overpriced more. Their mispricing measure for a stock is the average of its ranking percentiles across the 11 anomalies. We obtain these rankings from Jianfeng Yu's website.[2]

Each month we sort stocks into five quintiles on the basis of the Stambaugh, Yu and Yuan (2015) mispricing measure and into five quintiles using idiosyncratic volatility. This generates 25 double-sort portfolios. The rankings are independent, so the portfolios do not contain equal numbers of stocks. We then calculate value-weighted Fama-French-Carhart alphas for each portfolio each month. Panel A of Table 10 reports average monthly alphas for each portfolio over 1986-2016.

Our sample period is much shorter than the 1965 – 2011 sample period used in Stambaugh, Yu, and Yuan. Nevertheless, our results are very similar. For each IVOL quintile, underpriced stocks have higher alphas than overpriced stocks. The differences in alphas are statistically significant for all of the IVOL quintiles except the lowest. Moreover, as Stambaugh, Yu, and Yuan suggest, the difference is especially large for high IVOL stocks. The difference in alphas between under and over priced stocks is 2.138% per month for high IVOL stocks but just 0.13% for low IVOL stocks. Stambaugh, Yu, and Yuan attribute this to arbitrage risk slowing the price adjustment of high IVOL stocks. We also find that within the quintile of underpriced stocks high IVOL stocks outperform low IVOL stocks by 42 basis points per month. Among overpriced stocks, on the other hand, the high IVOL portfolio underperforms the low IVOL portfolio by 1.587%. These findings are consistent with Stambaugh, Yu, and Yuan's assertion that

[2] The data are updated by the end of year 2016, so our sample period in this section ends in 2016 rather than 2018.

high IVOL represents arbitrage risk and slows adjustment of prices both when stocks are overpriced and when they are underpriced.

The Markit data used in this paper starts in 2003. So, in Panel B of Table 10 we present Fama-French-Carhart alphas for the 25 portfolios using just the 2003-2016 period. In this abbreviated sample period, underpriced stocks still outperform overpriced stocks, and the difference increases with IVOL. These results are similar to the results for the much longer sample period of Stambaugh, Yu, and Yuan (2015), but the differences are smaller and less significant. Within the quintile of overpriced stocks, alphas decrease with IVOL. The high IVOL quintile of overpriced stocks has an alpha of -0.971% per month and the lowest IVOL portfolio has an alpha of -0.278% per month. The difference in the alphas is significant at the five percent level with a t-statistic of -1.97. In the underpriced quintile, high IVOL stocks still outperform low IVOL stocks as Stambaugh, Yu and Yuan predict, but the difference is statistically insignificant in the abbreviated 2003-2016 sample. The differences in alphas between under and overpriced stocks are large (74.3 basis points and 1.234%) and significant for two highest IVOL quintiles.

Both IVOL and underpricing may be related to the difficulty in borrowing shares. We calculate the mean indicative fee for stocks in each IVOL and mispricing portfolio. Panel C of Table 10 reports the mean values across months of 2003-2016 for each portfolio. For each mispricing quintile, there is a monotonic increase in indicative fees from the low to the high IVOL portfolios. So, for example, the mean indicative fee is 50 basis points per year for underpriced stocks in the lowest IVOL quintile, but 1.1% for underpriced stocks in the high IVOL quintile. The increase is largest, from 0.7% to 3.0% for overpriced stocks. Within IVOL quintiles, indicative fees increase monotonically from the underpriced quintile to the overpriced quintile. The increase in indicative fees is especially large, 1.1% to 3.0% for high IVOL stocks.

Hard-to-borrow stocks are defined as having no indicative fees, or indicative fees that are among the highest 20% for that month. We calculate the percentage of stocks that are hard-to-borrow each month for each portfolio over 2003-2016. Panel D of Table 10 reports the average, over all months, of the percentage of stocks in each of the 25 portfolios that are hard-to-borrow. For each mispricing quintile, whether stocks are under or overpriced, the percentage of stocks that are hard-to-borrow increases with IVOL. For example, among the underpriced stocks, 14.3% of the low IVOL stocks are hard to borrow and 33.7% of the high IVOL stocks are hard-to-borrow. For each IVOL quintile, the percentage of stocks that are hard-to-borrow increases monotonically from the underpriced to the overpriced quintile.

Panel E of Table 10 reports the alphas of the 25 IVOL and mispricing portfolios after removing hard-to-borrow stocks. There is no longer a significant difference between alphas of high and low IVOL portfolios within any mispricing category. Even among overpriced stocks, there is no difference in the alphas of high and low IVOL stocks after removing hard-to-borrow stocks. In Panel B, where hard-to-borrow stocks are included, the difference in alphas between high and low IVOL stocks is -0.693 in the overpriced stocks quintile. The difference has a t-statistic of -1.97. When the hard-to-borrow stocks are omitted, as shown in Panel E, the difference shrinks to -0.174 with a t-statistic of -0.49. The difference in alphas between over and underpriced stocks within IVOL quintiles is diminished when hard-to-borrow stocks are removed from the portfolios. The differences for the 2nd and 4th IVOL quintiles remain significant, but the difference for the high IVOL portfolio goes from -1.234 with a t-statistic of -3.32 in Panel B to -0.336 with a t-statistic of -0.96 in Panel E.

To summarize, with our short sample period we are unable to replicate all of the results of Stambaugh, Yu and Yuan (2015). The positive relation between IVOL and returns for underpriced stocks that they document is there but is insignificant using just 2003-2016. We are, however, able to replicate some of their results. The difference in returns between under and overpriced stocks is significant in our sample period, but disappears for high IVOL stocks once hard-to-borrow stocks are removed. The relation between IVOL and returns for overpriced stocks also disappears when hard-to-borrow stocks are omitted from the portfolios.

5. Lottery stocks, borrowing costs, and returns

If investors place a high value on large returns, they may be willing to accept smaller average returns for the possibility of earning a very large return. Bali, Cakici and Whitelaw (2011) suggest that the maximum return earned by a stock over a period may be a measure of the stock's attractiveness to lottery investors.

Each month over 2003-2018, we rank stocks on the basis of the largest daily return earned during that month. We then sort stocks into quintiles by largest return ranking. If investors prefer lottery stocks we would expect portfolios with large maximum returns to earn smaller average returns the next month. Average returns and four-factor alphas for the month after portfolio formation are provided in Table 11.

The first two columns show returns and alphas of the maximum return portfolios when the portfolios are equal-weighted. The low maximum return portfolio earns a return of 95.5 basis points the next month. The four factor alpha is 36.2 basis points. The t-statistic of 5.40 indicates the alpha is statistically significant at any conventional level. In contrast, the high maximum return portfolio earns an average return of 53.6 basis points and a four factor alpha of -47.6 basis points. The difference in returns between the high and low maximum return portfolios is -45.9 basis points and the difference in alphas is -83.8 basis points with a t-statistic of -4.37.

Hou and Loh (2016) show that the maximum return is highly correlated with idiosyncratic volatility, so we would expect the high maximum return portfolio to contain a lot of hard-to-borrow stocks. In the next two columns of Table 11 we provide average returns and alphas for the maximum return portfolios after removing hard-to-borrow stocks. When the hard-to-borrow stocks are removed, the high maximum return portfolio has larger returns than the low maximum return portfolio. The difference in alphas shrinks from 83.8 to 17.4 basis points and is now insignificant.

The last four columns of Table 11 repeat the first four using value-weighted portfolios. When all stocks are included, there is almost no difference in the returns of the high and low maximum return portfolios; just 1.6 basis points per month. The difference in alphas is -47.3 basis points with a t-statistic of -1.85. When the hard-to-borrow stocks are removed the difference in alphas declines to -36 basis points with a t-statistic of -1.57.

Lottery stocks, defined as those with high maximum returns, tend to be stocks with high idiosyncratic volatilities. Like high IVOL stocks, lottery stocks tend to be hard-to-borrow. And, like high IVOL stocks, the poor returns of lottery stocks are not statistically significantly different from zero after omitting hard-to-borrow stocks.

6. Conclusions

Stocks with high idiosyncratic volatility earn smaller risk-adjusted returns than stocks with low idiosyncratic volatility. Risk seems unlikely to explain these results. If investors hold diversified portfolios, idiosyncratic volatility shouldn't matter. If they hold portfolios that are not diversified, higher idiosyncratic risk should be associated with higher, not lower returns.

In this paper, we show that stocks with high idiosyncratic volatility tend to be stocks that are expensive to borrow and difficult to short. We show that when we remove hard-to-borrow stocks, there is no relation between idiosyncratic volatility and returns. The relation between idiosyncratic volatility and returns appears among stocks that are hard-to-borrow, but in this group the high volatility stocks are much harder to borrow than the low volatility stocks. As such, it is difficult to profit from the inverse relation between idiosyncratic volatility and returns.

We create portfolios matched on IVOL but with very different indicative fees. The high fee portfolio significantly underperforms the low fee portfolio. On the other hand, when we form portfolios matched on indicative fees but with very different IVOLs, the difference between alphas of high and low IVOL portfolios is small and insignificant. It appears that the relation between idiosyncratic volatility and returns is primarily a relation between ability to short and returns.

Table 1.
Summary Statistics
The sample period is 2003-2018. Idiosyncratic volatility (IVOL) is estimated monthly for each stock as the standard deviation of the residuals from the regression of the stock's return on the Fama-French factors. Total volatility (TVOL) is estimated monthly and is the standard deviation of returns for each stock. Stock borrowing fees are obtained from Markit. Book values are from Compustat, institutional holdings (INST) as a percentage of shares outstanding are from 13F filings obtained from WRDS, dispersion of analyst earnings forecasts (ADisp) is from IBES.

Panel A. Percentage of hard-to-borrow stocks in idiosyncratic volatility quintile portfolios.

	IVOL	Percent Missing Fee	Percent High Fee	Percent High or Missing Fee	Size ($ million)
Low	0.0086	16.39%	6.31%	21.72%	$12,235
2	0.0140	17.01%	8.12%	24.05%	$5,051
3	0.0196	19.19%	12.58%	30.18%	$2,539
4	0.0278	24.19%	21.97%	42.41%	$1,245
High	0.0540	33.60%	41.38%	62.45%	$513

Panel B. Percentage of hard-to-borrow stocks in total volatility quintile portfolios.

	Total Vol.	Percent Missing Fee	Percent High Fee	Percent High or Missing Fee	Size ($ million)
Low	0.0125	18.33%	9.27%	25.87%	$11,035
2	0.0182	17.19%	9.06%	24.86%	$5,567
3	0.0234	18.98%	12.51%	29.62%	$2,928
4	0.0310	23.27%	20.32%	40.11%	$1,459
High	0.0559	32.62%	39.21%	60.35%	$596

Panel C. Distribution of Indicative Stock Borrowing Fees by year.

Year	Number Stocks	% with fee	Mean Fee	Std. Dev.	1%	Median	80%	99%
2003	5,129	62.7%	0.0070	0.0121	0.0025	0.0050	0.0063	0.0600
2004	5,113	69.4%	0.0075	0.0126	0.0025	0.0050	0.0056	0.0700
2005	5,025	75.0%	0.0101	0.0209	0.0038	0.0048	0.0053	0.1000
2006	4,967	76.4%	0.0123	0.0263	0.0038	0.0047	0.0050	0.1229
2007	4,953	78.3%	0.0127	0.0264	0.0038	0.0045	0.0055	0.1200
2008	4,605	79.4%	0.0137	0.0331	0.0038	0.0045	0.0085	0.1538
2009	4,161	81.0%	0.0102	0.0344	0.0025	0.0033	0.0047	0.1665
2010	4,141	81.3%	0.0139	0.0516	0.0025	0.0038	0.0050	0.2423
2011	4,011	83.2%	0.0183	0.0760	0.0035	0.0038	0.0059	0.3304
2012	3,845	84.8%	0.0176	0.0645	0.0036	0.0038	0.0072	0.2848
2013	3,839	85.9%	0.0163	0.0601	0.0036	0.0038	0.0065	0.2818
2014	3,947	86.8%	0.0183	0.0610	0.0036	0.0038	0.0085	0.3000
2015	3,934	88.8%	0.0208	0.0663	0.0036	0.0038	0.0134	0.3068
2016	3,813	90.9%	0.0267	0.0829	0.0036	0.0041	0.0209	0.3762
2017	3,783	94.3%	0.0358	0.1159	0.0034	0.0039	0.0236	0.6583
2018	3,790	97.3%	0.0407	0.1457	0.0023	0.0033	0.0242	0.9030

Panel D. Distribution of indicative stock borrowing fees for stocks with first fees in that year.

Year	Number Stocks	Mean	Standard Deviation	1st Percentile	Median	99th Percentile
2003	3,217	0.0075	0.0108	0.0025	0.0050	0.0450
2004	546	0.0107	0.0122	0.0025	0.0050	0.0800
2005	476	0.0220	0.0246	0.0038	0.0100	0.1000
2006	318	0.0349	0.0380	0.0038	0.0198	0.1750
2007	379	0.0304	0.0287	0.0038	0.0201	0.1100
2008	212	0.0238	0.0283	0.0038	0.0093	0.1300
2009	148	0.0283	0.0904	0.0026	0.0063	0.2709
2010	212	0.0383	0.1053	0.0027	0.0074	0.5136
2011	205	0.0457	0.1259	0.0037	0.0085	0.6944
2012	202	0.0348	0.0693	0.0037	0.0131	0.3738
2013	270	0.0320	0.0665	0.0038	0.0099	0.4472
2014	329	0.0325	0.0794	0.0036	0.0146	0.2929
2015	275	0.0528	0.1088	0.0037	0.0163	0.6143
2016	256	0.0744	0.1484	0.0037	0.0245	0.8750
2017	308	0.1049	0.2047	0.0038	0.0351	1.1250
2018	335	0.1168	0.2415	0.0024	0.0316	1.1609

Panel E. Means and Standard deviations of variables used in the paper.

	IVol	TVol	Inst	ADisp	IndFee	Turnovr	Ln(Size)	Ln(M/B)	$R_{t-12,t-1}$
Mean	0.025	0.028	0.593	0.030	0.018	0.180	13.09	7.671	0.172
σ	0.024	0.025	0.322	0.306	0.068	0.050	2.017	1.036	0.762
Obs per Month	3,978	3,978	3,289	2,353	3,100	3,978	3,978	3,928	3,742

Panel F. Correlations of variables used in the paper.

	IVol	TVol	Inst	ADisp	IndFee	Turnovr	Ln(Size)	Ln(M/B)	$R_{t-12,t-1}$
IVol	1.000	0.971	-0.232	0.099	0.209	0.280	-0.371	-0.026	-0.057
TVol		1.000	-0.170	0.082	0.185	0.288	-0.318	-0.033	-0.066
Inst			1.000	-0.094	-0.234	0.216	0.613	0.156	-0.006
ADisp				1.000	0.118	0.032	-0.108	0.094	-0.011
IndFee					1.000	0.167	-0.206	0.025	-0.024
Turnovr						1.000	0.075	0.075	0.056
Ln(Size)							1.000	0.262	0.073
Ln(M/B)								1.000	0.208
$R_{t-12,t-1}$									1.000

Table 2.
Mean returns and Fama-French-Carhart Alphas for portfolios formed on idiosyncratic volatility. Idiosyncratic volatilities are estimated for each stock each month over 2003-2018 as the standard deviation of the residuals from a regression of the daily stock returns on the Fama-French Factors. Stocks are placed in five quintile portfolios based on IVOL each month. Returns and Fama-French-Carhart four-factor alphas are calculated for the following month for each portfolio. T-statistics, reported in parentheses below the mean alphas, are calculated using Newey-West standard errors adjusted for three lags.

Panel A. Equal-weighted portfolios.

	All Stocks		Omit if Missing Fee		Omit if Missing Fee or Highest 20% Fee		Include if Missing or Highest 20% Fee	
	Return	Alpha	Return	Alpha	Return	Alpha	Return	Alpha
Low	0.942	0.284	0.926	0.265	0.942	0.277	0.914	0.369
		(5.82)		(4.98)		(4.92)		(5.14)
2	0.978	0.161	1.020	0.207	0.994	0.188	0.802	0.056
		(2.53)		(3.50)		(3.39)		(0.49)
3	0.994	0.108	1.093	0.213	1.106	0.234	0.660	-0.165
		(1.57)		(3.36)		(4.24)		(-1.30)
4	0.982	0.004	1.083	0.130	1.136	0.188	0.534	-0.434
		(0.04)		(1.36)		(2.33)		(-2.55)
High	0.413	-0.584	0.874	-0.141	1.235	0.222	-0.191	-1.193
		(-2.83)		(-0.78)		(1.49)		(-4.69)
High-Low	-0.529	-0.868	-0.052	-0.406	0.293	-0.055	-1.105	-1.563
		(-3.84)		(-1.98)		(-0.31)		(-6.12)

Panel B. Value-weighted portfolios.

	All Stocks		Omit if Missing Fee		Omit if Missing Fee or Highest 20% Fee		Include if Missing or Highest 20% Fee	
	Return	Alpha	Return	Alpha	Return	Alpha	Return	Alpha
Low	0.752	0.107	0.772	0.142	0.786	0.161	0.651	-0.114
		(1.94)		(2.39)		(2.69)		(-0.82)
2	0.749	-0.082	0.748	-0.063	0.734	-0.033	0.624	-0.208
		(-1.28)		(-0.96)		(-0.51)		(-1.44)
3	0.791	-0.149	0.810	-0.089	0.815	-0.074	0.753	-0.257
		(-1.44)		(-0.83)		(-0.72)		(-1.34)
4	0.863	-0.174	0.983	-0.010	1.005	0.025	0.085	-1.031
		(-0.87)		(-0.07)		(0.17)		(-3.46)
High	0.635	-0.501	0.837	-0.301	1.061	-0.024	-0.222	-1.449
		(-2.18)		(-1.39)		(-0.10)		(-5.06)
High-Low	-0.116	-0.607	0.065	-0.443	0.275	-0.184	-0.873	-1.335
		(-2.28)		(-1.75)		(-0.69)		(-3.93)

Table 3.

Mean returns and Fama-French-Carhart Alphas for portfolios formed on total volatility.

Total volatilities are estimated for each stock each month over 2003-2018 as the standard deviation of the daily stock returns. Stocks are placed in five quintile portfolios based on total volatility each month. Returns and Fama-French-Carhart four-factor alphas are calculated for the following month for each portfolio. T-statistics, reported in parentheses below the mean alphas, are calculated using Newey-West standard errors adjusted for three lags.

Panel A. Equal-weighted portfolios.

	All Stocks		Omit if Missing Fee		Omit if Missing Fee or Highest 20% Fee		Include if Missing or Highest 20% Fee	
	Return	Alpha	Return	Alpha	Return	Alpha	Return	Alpha
Low	0.949	0.367	0.939	0.352	0.946	0.359	0.878	0.387
		(5.33)		(4.81)		(4.87)		(4.18)
2	0.988	0.206	1.019	0.242	1.031	0.252	0.867	0.126
		(2.82)		(3.44)		(3.85)		(0.97)
3	0.995	0.105	1.023	0.146	1.040	0.172	0.626	-0.195
		(1.40)		(2.12)		(2.61)		(-1.28)
4	0.925	-0.074	1.125	0.136	1.183	0.206	0.457	-0.524
		(-0.68)		(1.61)		(3.11)		(-2.94)
High	0.457	-0.630	0.889	-0.201	1.214	0.121	-0.221	-1.292
		(-3.29)		(-1.20)		(0.87)		(-5.23)
High-Low	-0.492	-0.997	-0.050	-0.553	0.267	-0.238	-1.098	-1.679
		(-4.80)		(-2.85)		(-1.39)		(-7.12)

Panel B. Value-weighted portfolios.

	All Stocks		Omit if Missing Fee		Omit if Missing Fee or Highest 20% Fee		Include if Missing or Highest 20% Fee	
	Return	Alpha	Return	Alpha	Return	Alpha	Return	Alpha
Low	0.815	0.236	0.828	0.260	0.820	0.260	0.684	-0.002
		(3.64)		(3.91)		(3.56)		(-0.02)
2	0.695	-0.107	0.761	-0.028	0.781	0.019	0.671	-0.169
		(-1.51)		(-0.37)		(0.27)		(-1.00)
3	0.816	-0.148	0.753	-0.179	0.783	-0.114	0.693	-0.262
		(-1.58)		(-1.68)		(-1.12)		(-1.66)
4	0.773	-0.256	0.964	-0.046	0.983	-0.023	0.093	-1.019
		(-1.47)		(-0.28)		(-0.16)		(-4.57)
High	0.742	-0.469	0.899	-0.283	0.942	-0.207	-0.338	-1.708
		(-1.98)		(-1.30)		(-0.91)		(-5.08)
High-Low	-0.073	-0.705	0.071	-0.543	0.122	-0.467	-1.022	-1.706
		(-2.54)		(-2.08)		(-1.68)		(-4.42)

Table 4.

Robustness tests

In each panel, stocks are sorted into quintiles by idiosyncratic volatility, calculated as the standard deviation of residuals from a regression of stocks returns on the Fama-French factors. Returns and four-factor alphas are calculated for each portfolio for the following month. T-statistics, reported in parentheses below the mean alphas, are calculated using Newey-West standard errors adjusted for three lags.

Panel A. Subperiod returns and alphas for equal-weighted portfolios.

	All Stocks				Omit if Missing Fee or Highest 20% Fee			
	2003-2009		2010-2018		2003-2009		2010-2018	
IVOL	Return	Alpha	Return	Alpha	Return	Alpha	Return	Alpha
Low	0.664	0.206	1.159	0.326	0.651	0.180	1.171	0.319
		(2.85)		(4.70)		(2.39)		(4.05)
2	0.721	0.054	1.185	0.265	0.812	0.141	1.202	0.261
		(0.54)		(4.61)		(1.65)		(3.96)
3	0.824	0.026	1.129	0.164	0.904	0.103	1.261	0.261
		(0.23)		(2.61)		(1.16)		(4.15)
4	1.096	0.141	0.891	-0.126	1.191	0.218	1.240	0.233
		(0.82)		(-0.92)		(1.50)		(2.08)
High	0.704	-0.353	0.190	-0.745	1.307	0.185	1.143	0.199
		(-1.25)		(-2.64)		(0.79)		(0.92)
High-	0.040	-0.558	-0.970	-1.071	0.657	0.005	-0.028	-0.120
Low		(-1.97)		(-3.30)		(0.02)		(-0.47)

Panel B. Subperiod returns and alphas for value-weighted portfolios.

	All Stocks				Omit if Missing Fee or Highest 20% Fee			
	2003-2009		2010-2018		2003-2009		2010-2018	
IVOL	Return	Alpha	Return	Alpha	Return	Alpha	Return	Alpha
Low	0.388	0.112	1.039	0.063	0.443	0.174	1.034	0.070
		(1.73)		(1.03)		(2.66)		(1.06)
2	0.379	-0.120	1.028	-0.053	0.413	-0.073	1.048	-0.037
		(-1.63)		(-0.49)		(-0.78)		(-0.31)
3	0.637	-0.026	0.925	-0.182	0.691	0.065	0.932	-0.168
		(-0.18)		(-1.51)		(0.37)		(-1.33)
4	0.786	-0.071	0.922	-0.176	1.084	0.236	1.081	-0.004
		(-0.25)		(-1.05)		(0.91)		(-0.02)
High	0.689	-0.388	0.579	-0.499	1.246	0.171	0.973	-0.102
		(-1.29)		(-1.81)		(0.50)		(-0.38)
High-	0.301	-0.501	-0.460	-0.562	0.803	-0.003	-0.061	-0.173
Low		(-1.48)		(-1.81)		(-0.01)		(-0.57)

Panel C. Portfolio returns and alphas after omitting hard-to-borrow stocks when hard-to-borrow is defined as among the top 20% of fees and when it is defined as having indicative fees at least the times the median across stocks that month.

	Equal-Weighted Portfolios				Value-Weighted Portfolios			
	Omit if Missing or Highest 20% Fee		Omit if Missing or Fee > 3 x Median		Omit if Missing or Highest 20% Fee		Omit if Missing or Fee > 3 x Median	
IVOL	Return	Alpha	Return	Alpha	Return	Alpha	Return	Alpha
Low	0.942	0.277	0.930	0.248	0.786	0.161	0.772	0.137
		(4.92)		(4.58)		(2.69)		(2.36)
2	0.994	0.188	1.030	0.197	0.734	-0.033	0.764	-0.063
		(3.39)		(3.34)		(-0.51)		(-0.89)
3	1.106	0.234	1.109	0.206	0.815	-0.074	0.829	-0.095
		(4.24)		(3.35)		(-0.72)		(-0.84)
4	1.136	0.188	1.220	0.233	1.005	0.025	1.059	0.055
		(2.33)		(2.33)		(0.17)		(0.31)
High	1.235	0.222	1.245	0.239	1.061	-0.024	1.057	-0.065
		(1.49)		(1.36)		(-0.10)		(-0.27)
High-Low	0.293	-0.055	0.315	-0.010	0.275	-0.184	0.285	-0.203
		(-0.31)		(-0.05)		(-0.69)		(-0.74)

Panel D. Returns and four factor alphas of equal-weighted portfolios of hard-to-borrow stocks when hard-to-borrow is defined in different ways.

	Missing or Highest 20% Fee		Missing		Highest 20% Fee		Missing or Fee > 3 x Median		Fee > 3 x Median	
IVOL	Return	Alpha	Return	Alpha	Return	Alpha	Return	Alpha	Return	Alpha
Low	0.914	0.369	0.894	0.311	0.909	0.486	0.963	0.418	1.061	0.627
		(5.14)		(3.12)		(4.44)		(5.49)		(4.41)
2	0.802	0.056	0.639	-0.152	1.057	0.391	0.830	0.079	1.117	0.441
		(0.49)		(-1.25)		(2.51)		(0.68)		(2.21)
3	0.660	-0.165	0.488	-0.392	0.910	0.147	0.648	-0.188	0.766	0.013
		(-1.30)		(-2.20)		(0.86)		(-1.53)		(0.07)
4	0.534	-0.434	-0.164	-1.164	0.959	0.006	0.551	-0.412	1.036	0.079
		(-2.55)		(-4.70)		(0.03)		(-2.48)		(0.39)
High	-0.191	-1.193	-1.151	-2.137	0.225	-0.792	-0.226	-1.225	-0.021	-1.032
		(-4.69)		(-6.03)		(-3.02)		(-4.92)		(-3.93)
High-Low	-1.105	-1.563	-2.046	-2.448	-0.684	-1.278	-1.189	-1.643	-1.082	-1.659
		(-6.12)		(-6.61)		(-5.05)		(-6.45)		(-5.93)

Panel E. Returns and four factor alphas of value-weighted portfolios of hard-to-borrow stocks when hard-to-borrow is defined in different ways.

	Missing or Highest 20% Fee		Missing		Highest 20% Fee		Missing or Fee > 3 x Median		Fee > 3 x Median	
IVOL	Return	Alpha	Return	Alpha	Return	Alpha	Return	Alpha	Return	Alpha
Low	0.651	-0.114	0.677	-0.075	0.829	0.176	0.656	-0.105	1.101	0.437
		(-0.82)		(-0.52)		(0.82)		(-0.73)		(1.68)
2	0.624	-0.208	0.551	-0.273	0.989	0.151	0.632	-0.209	1.091	0.217
		(-1.44)		(-2.10)		(0.67)		(-1.63)		(0.75)
3	0.753	-0.257	0.623	-0.450	0.842	-0.089	0.677	-0.349	0.840	-0.122
		(-1.34)		(-2.05)		(-0.36)		(-1.84)		(-0.40)
4	0.085	-1.031	-0.220	-1.403	0.354	-0.646	0.202	-0.907	0.595	-0.425
		(-3.46)		(-3.42)		(-2.55)		(-2.96)		(-1.50)
High	-0.222	-1.449	-0.627	-1.780	-0.117	-1.397	-0.208	-1.399	-0.319	-1.557
		(-5.06)		(-5.17)		(-4.33)		(-4.41)		(-3.97)
High-Low	-0.873	-1.335	-1.304	-1.705	-0.946	-1.573	-0.864	-1.294	-1.420	-1.995
		(-3.93)		(-4.09)		(-3.92)		(-3.38)		(-3.84)

Table 5.
Fama-French-Carhart abnormal returns for portfolios sorted on idiosyncratic volatility and indicative fees. Each month over 2003-2018, we double sort stocks into quintiles based on idiosyncratic volatility and into terciles based on indicative fees. Fama-French-Carhart abnormal returns are calculated for each of the 15 portfolios over the next month. The abnormal returns are then averaged over months from 2003-2018.

Panel A. Equal-weighted portfolios.

	Indicative Fee			
Idiosyncratic σ	Low	2	High	High - Low
Low	0.229	0.226	0.374	0.149
				(2.10)
2	0.185	0.079	0.315	0.094
				(0.79)
3	0.255	0.050	0.262	0.023
				(0.22)
4	0.180	0.407	0.022	-0.180
				(-1.22)
High	0.378	0.151	-1.100	-1.500
				(-5.97)
High - Low	0.155	-0.055	-1.472	
	(0.75)	(-0.22)	(-5.10)	

Panel B. Value-weighted portfolios

	Indicative Fee			
Idiosyncratic σ	Low	2	High	High - Low
Low	0.127	0.073	0.262	0.135
				(1.02)
2	-0.098	-0.028	0.016	0.075
				(0.44)
3	-0.072	0.081	-0.239	-0.185
				(-0.71)
4	0.064	-0.045	-0.415	-0.505
				(-1.75)
High	0.102	0.038	-1.758	-1.822
				(-4.79)
High - Low	-0.054	-0.034	-1.997	
	(-0.19)	(-0.11)	(-4.92)	

Table 6.
Time series mean indicative fees and idiosyncratic volatility for portfolios of stocks formed by sorting into terciles by indicative fee and quintiles by idiosyncratic volatility

	Panel A. Indicative Fee			Panel B. Idiosyncratic Volatility		
	Indicative Fee					
Idiosyncratic σ	Low	2	High	Low	2	High
Low	0.004	0.004	0.012	0.0086	0.0087	0.0084
2	0.004	0.004	0.016	0.0138	0.0139	0.0140
3	0.004	0.004	0.027	0.0193	0.0194	0.0196
4	0.004	0.005	0.056	0.0273	0.0275	0.0280
High	0.004	0.011	0.125	0.0480	0.0511	0.0589

Table 7.

Returns of portfolios of stocks matched on idiosyncratic volatility with very different indicative fees, and matched on indicative fees with very different idiosyncratic volatilities.

Each month, we identify pairs of stocks with the following criteria. First, the idiosyncratic volatility of the first stock divided by the idiosyncratic volatility of the second stock must between 0.99 and 1.01. Second, the indicative fee of the first stock divided by the indicative fee of the second stock must be less than 0.5 or more than 2.0. The low indicative fee stock of the pair is placed in the low indicative fee portfolio for that month while the high indicative fee stock is placed in the high indicative fee portfolio for that month. No stock is used more than once during a month. An equal-weighted average return and four-factor alpha is calculated for each portfolio each month. The mean monthly return and alpha for 2003-2018 are hen calculated. An analogous procedure is used to separate stocks into high and low idiosyncratic volatility portfolios each month while keeping indicative fees the same for both.

	Mean Num. In Portfolio	Mean Indicative Fee	Mean Idiosyncratic σ	Mean Return	Mean Four Factor Alpha
	Panel A. Controlled variables matched within 1%, high value is ≥ 2 times low value				
Low Fee	581.7	0.535%	3.030%	1.076%	0.091%
High Fee	581.7	6.535%	3.046%	0.684%	-0.208%
Difference		5.992%	0.016%	-0.392%	-0.300%
				(-3.44)	(-2.72)
Low IVOL	1,249.0	1.079%	1.246%	0.958%	0.189%
High IVOL	1,249.0	1.079%	3.293%	1.113%	0.091%
Difference		0.000%	2.048%	0.155%	-0.098%
				(0.88)	(-0.93)
	Panel B. Controlled variables matched within 0.75%, high value is ≥ 3 times low value				
Low Fee	469.0	0.537%	3.115%	1.058%	0.072%
High Fee	469.0	7.706%	3.128%	0.614%	-0.302%
Difference		7.169%	0.013%	-0.444%	-0.374%
				(-3.55)	(-3.05)
Low IVOL	733.5	1.001%	1.049%	0.938%	0.240%
High IVOL	733.5	1.001%	3.887%	1.134%	0.088%
Difference		0.000%	2.838%	0.196%	-0.152%
				(0.83)	(-1.10)

Table 8. Monthly Fama-MacBeth regressions of stock excess returns, defined as the difference between the stock return and the risk free rate on idiosyncratic volatility and other factors. Idiosyncratic σ is estimated over the previous month as the standard deviation of the residual from the regression of the daily stock return on the Fama-French three factors. Inst. Holding is the proportion of the firm's common stock held by institutions. ADISP is the percentage dispersion of analyst's forecasted earnings. Indicative fee is the annualized cost of borrowing shares obtained from Markit. $D_{InstMissing}$, $D_{DispMissing}$, and $D_{FeeMissing}$ are dummy variables that take values of one if these variables are missing for a stock during a month. Turnover is the volume divided by shares outstanding during the month. Size is the market capitalization of the stock and ME/BE is the ratio of the market to the book value of the stock the previous month. Ret_t is the stocks return the previous month while $Ret_{t-12, t-1}$is the stock return for the 12 months prior to that.

Panel A. Idiosyncratic volatility is an explanatory variable.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$D_{InstMissing}$			-0.011			-0.011	-0.008
			(-5.91)			(-6.55)	(-3.18)
$D_{DispMissing}$				-0.003		-0.002	-0.002
				(-2.16)		(-2.11)	(-1.95)
$D_{FeeMissing}$					-0.013		-0.007
					(-8.94)		(-3.44)
Idiosyncratic σ	-0.143	-0.085	-0.066	-0.083	-0.056	-0.066	-0.053
	(-3.06)	(-2.08)	(-1.65)	(-2.05)	(-1.41)	(-1.65)	(-1.36)
Inst. $Holding_t$			0.001			0.000	-0.002
			(0.73)			(0.25)	(-1.00)
$ADISP_t$				-0.011		-0.010	-0.009
				(-2.03)		(-1.77)	(-1.62)
Indicative Fee_t					-0.079		-0.080
					(-5.60)		(-5.60)
$Turnover_t$		-0.230	-0.229	-0.292	-0.134	-0.255	-0.132
		(-0.87)	(-0.93)	(-1.22)	(-0.51)	(-1.10)	(-0.58)
$Log(Size)_t$		-0.000	-0.001	-0.001	-0.001	-0.001	-0.001
		(-0.94)	(-1.40)	(-1.63)	(-2.26)	(-1.76)	(-1.89)
$Log(ME/BE)_t$		-0.002	-0.002	-0.002	-0.001	-0.002	-0.001
		(-2.72)	(-2.58)	(-2.76)	(-2.32)	(-2.59)	(-2.29)
Ret_t		-0.007	-0.008	-0.007	-0.009	-0.008	-0.009
		(-1.28)	(-1.58)	(-1.31)	(-1.66)	(-1.61)	(-1.86)
$Ret_{t-12, t-1}$		-0.001	-0.001	-0.000	-0.001	-0.001	-0.001
		(-0.28)	(-0.38)	(-0.14)	(-0.40)	(-0.27)	(-0.31)
Adj. R^2	0.005	0.029	0.032	0.031	0.034	0.036	0.036
Observations	3,960	3,709	3,709	3,709	3,709	3,709	3,709

Table 9

Mean options order imbalances (OOI) for hard-to-borrow and other stocks by IVOL portfolio.
Options order imbalances are calculated for each stock each month by summing the product of the option volume by the option delta over all trades. The options order imbalance is then standardized by dividing by shares outstanding. Mean OOIs are calculated across stocks in each portfolio and are reported as a percentage of shares outstanding.

	All Hard-to-Borrow		Missing Fees		Top 20% Fees		Not Hard-to-Borrow	
IVOL	Indic. Fee	Mean OOI	Indic. Fee	Mean OOI	Indic. Fee	Mean OOI	Indic. Fee	Mean OOI
Low	0.037	-0.0043	.	-0.0032	0.037	-0.0100	0.004	-0.0024
2	0.043	-0.0070	.	-0.0027	0.043	-0.0194	0.004	-0.0027
3	0.055	-0.0098	.	-0.0036	0.055	-0.0194	0.004	-0.0009
4	0.075	-0.0088	.	-0.0014	0.075	-0.0163	0.005	0.0013
High	0.108	-0.0143	.	-0.0082	0.108	-0.0192	0.005	0.0077
H - L		-0.0100		-0.0050		-0.0092		0.0101
		(-6.21)		(-2.54)		(-2.24)		(14.40)

Table 10.

Idiosyncratic volatility, mispricing, and hard-to-borrow stocks.

Each month stocks are sorted into five quintiles on the basis of an index of 11 variables shown to indicate mispricing with each quintile then sorted into five quintiles based on idiosyncratic volatility as in Stambaugh, Yu, and Yuan (2015). Fama-French-Carhart four factor alphas are computed for each value-weighted portfolio each month. Mean monthly alphas are reported with t-statistics in parentheses underneath. They are calculated with Newey-West standard errors using three lags.

Panel A. 1986-2016

	Underpriced	2	3	4	Overpriced	Over-Under
Low IVOL	0.083	0.231	0.094	-0.046	-0.047	-0.130
	(0.91)	(2.78)	(1.03)	(-0.40)	(-0.30)	(-0.75)
2	0.053	0.045	-0.139	-0.048	-0.493	-0.546
	(0.58)	(0.42)	(-1.40)	(-0.41)	(-3.03)	(-3.10)
3	0.312	-0.130	-0.100	-0.140	-0.531	-0.843
	(2.54)	(-1.04)	(-0.71)	(-1.01)	(-3.04)	(-3.78)
4	0.325	0.020	0.149	-0.310	-0.854	-1.179
	(2.02)	(0.12)	(1.89)	(-1.70)	(-4.85)	(-5.09)
High IVOL	0.504	0.013	0.002	-0.606	-1.634	-2.138
	(2.12)	(0.06)	(0.02)	(-2.79)	(-8.17)	(-7.05)
High - Low	0.421	-0.219	-0.304	-0.561	-1.587	
	(1.60)	(-0.89)	(-1.98)	(-2.23)	(-5.59)	

Panel B. 2003-2016

	Underpriced	2	3	4	Overpriced	Over-Under
Low IVOL	0.082	0.221	0.055	0.107	-0.278	-0.360
	(0.89)	(2.34)	(0.50)	(0.72)	(-1.43)	(-1.57)
2	-0.007	-0.001	-0.001	0.084	-0.523	-0.516
	(-0.06)	(-0.00)	(-0.01)	(0.43)	(-2.34)	(-1.93)
3	-0.036	-0.324	-0.141	-0.151	-0.253	-0.218
	(-0.29)	(-1.82)	(-0.82)	(-0.73)	(-1.07)	(-0.78)
4	0.034	-0.011	-0.160	-0.411	-0.709	-0.743
	(0.18)	(-0.05)	(-0.66)	(-1.36)	(-2.67)	(-2.22)
High IVOL	0.263	0.033	-0.185	-0.457	-0.971	-1.234
	(0.98)	(0.11)	(-0.63)	(-1.69)	(-3.39)	(-3.32)
High - Low	0.181	-0.188	-0.240	-0.564	-0.693	
	(0.57)	(-0.58)	(-0.75)	(-1.89)	(-1.97)	

Panel C. Indicative Fee Values

	Underpriced	2	3	4	Overpriced
Low IVOL	0.005	0.005	0.006	0.006	0.007
2	0.005	0.006	0.006	0.007	0.007
3	0.006	0.007	0.008	0.008	0.010
4	0.008	0.008	0.009	0.010	0.014
High IVOL	0.011	0.013	0.017	0.020	0.030

Panel D. Percentage of hard-to-borrow

	Underpriced	2	3	4	Overpriced
Low IVOL	0.143	0.194	0.225	0.242	0.264
2	0.180	0.204	0.221	0.244	0.279
3	0.207	0.216	0.242	0.265	0.310
4	0.249	0.256	0.283	0.307	0.361
High IVOL	0.337	0.338	0.379	0.408	0.488

Panel E. Take out hard-to-borrow

	Underpriced	2	3	4	Overpriced	Over-Under
Low IVOL	0.093	0.349	0.171	0.293	-0.029	-0.122
	(0.93)	(3.58)	(1.46)	(1.78)	(-0.14)	(-0.54)
2	0.017	0.034	-0.352	0.028	-0.500	-0.516
	(0.13)	(0.24)	(-2.22)	(0.14)	(-2.47)	(-2.02)
3	-0.078	0.064	-0.057	0.176	0.043	0.121
	(-0.60)	(0.41)	(-0.31)	(0.89)	(0.16)	(0.39)
4	0.199	-0.104	-0.083	-0.052	-0.570	-0.768
	(1.01)	(-0.48)	(-0.33)	(-0.25)	(-2.13)	(-2.39)
High IVOL	0.133	0.006	-0.084	-0.085	-0.203	-0.336
	(0.45)	(0.02)	(-0.30)	(-0.33)	(-0.69)	(-0.96)
High - Low	0.040	-0.344	-0.255	-0.377	-0.174	
	(0.11)	(-1.14)	(-0.81)	(-1.32)	(-0.49)	

Table 11.

Lottery stock returns and short selling constraints.

Each month over 2003-2018, stocks sorted into quintile portfolios on the basis of the previous month's maximum daily return. Alpha for the portfolios are calculated from the Fama-French-Carhart four factor model. Returns and alphas are the time series means over months from 2003 through 2018.

	Equal Weighted Portfolios				Value Weighted Portfolios			
	All Stocks		Omit if Missing Fee or Highest 20% Fee		All Stocks		Omit if Missing Fee or Highest 20% Fee	
	Return	Alpha	Return	Alpha	Return	Alpha	Return	Alpha
Low	0.955	0.362	1.024	0.376	0.854	0.254	0.864	0.288
		(5.40)		(5.08)		(4.05)		(3.90)
2	1.002	0.192	1.082	0.292	0.663	-0.125	0.766	0.011
		(3.17)		(5.06)		(-2.28)		(0.20)
3	0.946	0.045	1.039	0.149	0.735	-0.184	0.723	-0.162
		(0.66)		(2.52)		(-1.92)		(-1.81)
4	0.836	-0.141	1.061	0.097	0.712	-0.294	0.759	-0.193
		(-1.33)		(1.44)		(-1.90)		(-1.45)
High	0.536	-0.476	1.209	0.202	0.838	-0.218	0.953	-0.072
		(-2.61)		(1.58)		(-1.04)		(-0.41)
High-Low	-0.459	-0.838	0.186	-0.174	-0.016	-0.473	0.089	-0.360
		(-4.37)		(-1.11)		(-1.85)		(-1.57)

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